Math 131B-1: Homework 7

Due: February 26, 2014

1. Read Apostol Sections 1.21-27, 1.29, 1.32, 9.14-15.
2. Do Apostol problems 1.27, 1.30, 1.36.
3. Prove the inequalities for the complex norm from class, namely that
   - $|R(z)| \leq |z|$
   - $|I(z)| \leq |z|$
   - $|z| \leq |R(z)| + |I(z)|$
   - $|z + w| \leq |z| + |w|$
4. Prove that $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ and $\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.
5. Prove that if $\{z_n\}$ and $\{w_n\}$ are sequences of complex numbers such that $z_n \to z$ and $w_n \to w$, then $z_n w_n \to zw$.
6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function which is differentiable at x_0, with $f(x_0) = 0$ and $f'(x_0) \neq 0$. Show there exists a $c > 0$ such that $f(y) \neq 0$ for $0 < |x - y| < c$. In particular, show there is a $c > 0$ such that $\sin(x) \neq 0$ for $0 < x < c$.
7. Let $\tan(x) = \frac{\sin x}{\cos x}$. Show that the tangent function is differentiable and monotone increasing on $(\frac{-pi}{2}, \frac{\pi}{2})$, and thus invertible. Show (using what we proved about the derivatives of inverse functions last quarter) that the derivative of $g(x) = \tan^{-1}(x)$ is $\frac{1}{1+x^2}$.
8. Prove that a continuous 1-periodic function $f\colon \mathbb{R} \to \mathbb{C}$ is bounded. Give an example showing that if f is not assumed to be continuous, this need not be true.